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| The Trashcan (Software and Media Publication, NFP) |
| 2D Terrestrial (Nonturbulent) Gravitational Motion |
| Physics Supplement for Application to Software |

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| Ray Arias  10 March 2018 to 29 April 2018 |

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**I Need to Make a 2D Simulation, Game, or Other Software, and I Need to Account for Gravity! What Do I Do?**

Calm down. It’s not a problem. You just need to learn (or brush up on) your terrestrial kinematics. This is a basic Newtonian (as opposed to Relativistic or Quantum and all that fancy newer stuff) description of how gravity and motion work here on Earth (as opposed to in space and stuff). Terrestrial kinematics entails horizontal kinematics, which as you may guess is a description of horizontal motion, and vertical kinematics, a description of vertical motion. Here is each one in detail. If you don’t know calculus, don’t let it scare you. I just put it there for those who can read it.

**Horizontal Kinematics**

Since any forces pushing or pulling upon any object in the horizontal while it is flying through the air are negligible. (There is friction, both static and kinetic, with gaseous air particles, but it is very small for most objects, except when dealing with winds of great velocity, as in tornados and hurricanes, or taking into account objects that can be greatly affected by wind resistance or can take advantage of aerodynamic lift, such as gliders, airplanes, and parachutes. By the way, the physics of wind resistance and aerodynamic life are extremely complicated and are not covered in this short supplemental paper.) Therefore, a solid object being thrown, catapulted, or otherwise projected into the air, can generally be considered as having neither any thrust added to, nor any resistance taken away from, its initial motion. Hence, the horizontal speed is practically constant; there is no horizontal component of acceleration to take into account. Consequently, the horizontal position only changes by adding the same constant quantity to the previous position when animating in 2D software.

**Vertical Kinematics**

As with horizontal kinematics, the mechanics of vertical motion of objects through the air, though continuously affected by the friction of gaseous air particles, this friction is so small that in most circumstances that it can be neglected, again, except in cases where it becomes so large, such as in gliders, airplanes, parachutes, etc. With this in mind, the vertical motion of objects flying through the air, for instance after being catapulted or thrown, is similar to that of an object loaded on top of a coiled spring and then propelled by allowing the spring to uncoil. This object would then be shot upward by the force of the uncoiling spring. This upward force would immediately be counteracted by the downward force of terrestrial gravity. Once the object is out of contact with the spring, the object would continue upward with whatever velocity was given to it by the force of the spring. This upward velocity would reduce until it reaches zero as gravitation would be the only force affecting the object after that point. Additionally, after reaching its vertex (the highest point in its vertical path), the object would then gain a downward velocity from the continued gravitation affecting it. This velocity would be small at first and then grow to its maximum just before the object either reloads the spring, recoiling it in the process, or hits the ground. Generally, a vertically propelled object returns with a downward velocity of the same magnitude, at the same height that it started, as the initial upward velocity that it was originally driven to by the upward force of the spring. For example, if a ball was thrown straight up at 10 miles/hour, by the time it finishes going up, gets to it vertex, and goes down, when it reaches the same height it was thrown up at, it should be going straight down at pretty much (if not *exactly*) 10 mi/hr.

After this point, if an object with downward velocity is allowed to make contact with the ground, depending upon what the ground is made of and what the object is made of, the object will bounce back up at some fractional upward velocity of the downward velocity it came down with. For a rubber ball on a hard surface, this fraction is very large, for a less bouncy object on a plush surface, this fraction is very small. This fraction is known as the coefficient of restitution (CoR). However, other than touching on it briefly here, I will not cover CoR any further in this paper.

The terrestrial force of gravity upon any object’s mass is *G*⊕,which is equal to *mg*, the mass of the object (*m*) multiplied by the acceleration given to it by the gravitation between the object and the Earth (*g*). This acceleration is expressed either in US Customary Units as 32.1740 feet/sec2 or in the International System of Units (SI) as 9.80665 meters/sec2. These are averages and actual measurements for *g* can vary according to altitude from sea level.

If these facts are applied separately to each of the axes, when plotted on a graph using sample initial velocities for each direction, the following is obtained:

If both axes are applied jointly, this graph is obtained:

**OK, So What Does All This Mean for Simulations, Games, and Other Software?**

Well, I understand many people reading this may not even know calculus. (I just thought I’d throw the calculus in for those who do understand it.) But basically, this is going to all break down like this. Since on the horizontal axis, the velocity is going to be constant, you can assign a constant velocity

vx = 10;

x = x + vx;

or neglect it entirely as you can just hard code the constant directly when you are changing the x-coordinate (x = x + 10;). For the vertical velocity, you will need a variable though. It is your choice, however, if you want to keep a constant, such as,

g = 9.8;

vy = vy – g;

y = y + vy;

or hard code when you change your vertical velocity.

vy = vy – 9.8;

y = y + vy;

Nevertheless, keep in mind that 9.8 (m/s)/s—or whichever number you select for *g*—is a change that occurs every whole second, so if you are changing your vertical velocity 100 times per second, you will need to divide this number by 100. Thus, you should use either something like

g = 9.8;

vy = vy – (g / 100);

or,

g100 = 0.98;

vy = vy – g100;

or even,

g = 9.8;

dt = 100;

vy = vy – (g / dt);

Whichever works for you, when your object finally flies across the screen, you should get a similar shape to the second graph (concave down parabola). If not, you need to double check your math. Good luck!

**Oh, I Almost Forgot! Here’s a Little Software Library to Help You Out!**

I hope you find bundled with this document a software library I made in C++. This is to help you create objects that behave as if they were under the influence of terrestrial gravitation as well as in order to instruct you as to how to code such objects and the behavior underlying them appropriately. I call it, simply enough, TerrestrialGravitation.

By the way, I have decided that anyone who wants to use this commercially and proprietarily can either afford other software that would effectively do what this library already does, or can afford to effectively recode it from scratch by reverse engineering it, which they are more than welcome to do. This is why this library is released under harder CopyLeft license of GNU GPL, v3, rather than the lesser so LGPL.

These classes have constructors as fully defined as possible, from default constructors with no parameters, to fully defined constructors with every variable member defined as a parameter, to copy constructors, to assignment operators, to even destructors.

class Cartesian

The library starts with the class Cartesian. These are just objects that have integer x and y coordinates (int), as well as double precision floating point coordinates (double), designated as xx and yy. I made two sets of coordinates so that switching between doubles and ints would not be any excessive burden. For scientific precision, doubles are best used. However, for games and graphic simulations that require pixel coordinates, ints are much faster and convenient to use. Therefore, the Cartesian class has coordinates of both types.

Due to this, I inserted three functions to ease using objects that come out of this class. First is the Boolean function bool concordance(), which requires no parameters. This checks to see if the values of x and xx are within less than an integer unit’s difference equal. If this test passes, it checks for the same level of equality between y and yy. If this test passes, TRUE is returned, otherwise the result is FALSE. Next, is the void function concordanceInt(). Basically, all this does is force concordance, or agreement, between the ints and the doubles. In the case of concordanceInt(), this is accomplished by assigning the doubles to the values of the ints (casting them as doubles in the process, of course). Finally, the void function concordanceDouble() assigns the values of the doubles (casted as ints, and therefore, truncated) to the ints.

Notwithstanding the assignment operator (operator=(…)), I have also placed an addition operator (operator+(…)), as well as a subtraction operator (operator-(…)), in order to facilitate the addition and subtraction of coordinates of different Cartesian objects.

There are functions for finding distance from the Cartesian object to the origin, or between two Cartesian objects. This can be given as a double or an int, according to if you use distanceInt(…) or distanceDouble(…). Computing distance is done by using the Pythagorean theorem: *a*² + *b*² = *c*², or in this case, .

There are also functions that return the angle pointing to the origin or the other Cartesian object. The angleint(…)functions return degrees and the angleDouble(…)ones return radians. However, if you use angleInt(…), the doubles will be concordanceInt()ed with the ints, and if you use angleDouble(…), vice versa. Angles are calculated using the arctangent of *y* / *x*, or in C code: double thetaRad = atan(yy / xx); the Int version carries out this same calculation but multiplies the result by the constant 180/π, or again in C: int thetaDeg = (int) (180 / PI) \* atan(yy / xx); Here are the formulas:

There are functions for reading and assigning either x or y or either of the doubles xx or yy. After each assignment, one of the concordance assignment functions is executed so that agreement is maintained between ints and doubles.

Lastly, there is an atOrigin() function that assigns everything to zero (referred to on Cartesian graphs as the Origin, or (0, 0)). In order to facilitate assignment to another Cartesian object, atOrigin() returns the result as well as assigning it to the current object (\*this).

The class Cartesian, as you will see, is a freely accessible encapsulated class of class InertBody. All members are public, so they are freely accessible. However, two Cartesian objects (position and velocity) will be used in class InertBody. Also, all members of class Cartesian are accessible by way of functions (for instance, int X() and void X(int a) to read and assign the Cartesian member int x respectively, etc.). There are also functions to write members jointly (void XY(int a, int b) and void XXYY(double aa, double bb)).

*Variable Members:* ints: x, y, doubles: xx, yy.

*Constructors:* Cartesian(void): (***default constructor***) assigns 0 to x and y and 0.0F to xx and yy.

Cartesian(int a, b): assigns a to x and b to y and executes concordanceInt() to make the doubles agree with the ints.

Cartesian(double aa, bb): assigns aa to xx and bb to yy and executes concordanceDouble() to make the ints agree with the doubles.

Cartesian(const Cartesian &cartecopy): (***copy constructor***) takes each member of a given Cartesian object and assigns its value to each member of the current object.

*Operators:* ***Assignment*** (operator=(const Cartesian &carteassignment)): Same as ***copy constructor***, but checks for self-assignment, for which case nothing happens, and at the end the current object (\*this) is returned.

***Addition*** (operator+(const Cartesian &carte1, const Cartesian &carte2)): Adds the coordinates of two Cartesian objects and returns a third whose coordinates are the sum of the other two.

***Subtraction*** (operator–(const Cartesian &carte1, const Cartesian &carte2)): Subtracts the coordinates of a second Cartesian object from the coordinates of the first one and returns a third object whose coordinates are the difference of the other two.

*Destructor:* ~InertBody(void): deletes current Cartesian object.

*Functions for reading the current position:*

int X(void): returns the current object’s value for int x.

int Y(void): returns the current object’s value for int y.

double XX(void): returns the current object’s value for double xx.

double YY(void): returns the current object’s value for double yy.

Cartesian Position(void): returns a Cartesian object whose coordinates are identical to the current object’s coordinates.

*Functions for setting the current position:*

void X(int a): assigns a to int x, executes concordanceInt().

void Y(int b): assigns b to int y, executes concordanceInt().

void XY(int a, int b): assigns a to int x, assigns b to int y, executes concordanceInt().

void XX(double aa): assigns aa to double xx, executes concordanceDouble().

void YY(double bb): assigns bb to double yy, executes concordanceDouble().

void XXYY(double aa, double bb): assigns aa to double xx, assigns bb to double yy, executes concordanceDouble().

void Position(Cartesian pos): assigns pos’s coordinates to current object’s coordinates by way of assigning pos.xx to xx, assigning pos.yy to yy, and executing concordanceDouble().

*Functions that compute distance*:

int distanceInt(void): calculates distance from Cartesian object to origin (0, 0) to nearest integer unit and returns result.

double distanceDouble(void): calculates distance from Cartesian object to origin (0, 0) and returns result.

double distance(void): same as distanceDouble().

int distanceInt(int a, int b): calculates distance between from current Cartesian object’s coordinate to coordinates given by (a, b) to the nearest integer unit and returns the result.

int distance(int a, int b): same as distanceInt(int a, int b).

double distanceDouble(double aa double bb): calculates distance from current Cartesian object’s coordinates to coordinates given by (aa, bb) and returns the result.

double distance(double aa, double bb): same as distanceDouble(double aa, double bb).

int distanceInt(Cartesian carte): calculates the distance between the current Cartesian object and another Cartesian object given by carte to the nearest integer unit and returns the result.

double distanceDouble(Cartesian carte): calculates the distance between the current Cartesian object and another Cartesian object given by carte and returns the result.

double distance(Cartesian carte): same as distanceDouble(Cartesian carte).

*Functions for finding angles:*

int angleInt(void): computes the angle from the current Cartesian object to the origin (0, 0) in degrees to the nearest integer unit and returns the result.

double angleDouble(void): computes the angle from the current Cartesian object to the origin (0, 0) in radians and returns the result.

double angle(void): same as angleDouble(void).

int angleInt(int w, int z): computes the angle in degrees from the current Cartesian object to coordinates given by (w, z) to the nearest integer unit and returns the result.

int angle(int w, int z): same as angleInt(int w, int z).

double angleDouble(double ww, double zz): computes the angle in radians from the current Cartesian object to coordinates given by (ww, zz) and returns the result.

double angle(double ww, double zz): same as angleDouble(double ww, double zz).

int angleInt(Cartesian carte): calculates angle in degrees from current Cartesian object to another Cartesian object given by carte to the nearest integer unit and returns the result.

double angleDouble(Cartesian carte): calculates angle in radians from current Cartesian object to another Cartesian object given by carte and returns the result.

double angle(Cartesian carte): same as angleDouble(Cartesian carte).

*Functions for checking and forcing concordance of* int *and* double *coordinate sets:*

bool concordance(void): checks to see if (x, y) and (xx, yy) agree to the nearest integer unit. Returns TRUE if they do and FALSE if they don’t.

void concordanceInt(void): forces concordance, or agreement, between double and int coordinates, doubles are assigned double casted values of ints.

void concordanceDouble(void): forces concordance, or agreement, between double and int coordinates, ints assigned int casted values of doubles.

*Miscellaneous Function:*

Cartesian atOrigin(void): current Cartesian object is assigned to be at the origin (0, 0) and (0.0F, 0.0F) and is returned to facilitate assignment.

class InertBody

The class InertBody is an encapsulator class of class Cartesian in that class InertBody makes use of class Cartesian by creating two objects from it, Cartesian position and Cartesian velocity, that are members of this class. There is also a protected member, bool \_at\_rest, that every function that sets Cartesian velocity tests to be (0, 0) by way of declaring a new Cartesian Zero and then executing its atOrigin() function and then comparing velocity to Zero. This class only applies position and velocity, but neither any gravitation, nor any other force, hence the name of this class.

*Variable Members:* Cartesian objects: position, velocity.

**Protected** *Variable Member:* bool \_at\_rest.

*Constructors:* InertBody(void): (***default constructor***) executes Cartesian function atOrigin() for both position and velocity, sets bool \_at\_rest to TRUE.

InertBody(int a, int b): assigns (a, b) to Cartesian position, executes position.concordanceInt(), executes velocity.atOrigin(), and sets bool \_at\_rest to TRUE.

InertBody(double aa, double bb): assigns (aa, bb) to Cartesian position, executes position.concordanceDouble(), executes velocity.atOrigin(), and sets bool \_at\_rest to TRUE.

InertBody(Cartesian pos): assigns (pos.xx, pos.yy) to Cartesian position, executes position.concordanceDouble(), executes velocity.atOrigin(), and sets bool \_at\_rest to TRUE.

InertBody(int a, int b, int vx, int vy): assigns (a, b) to Cartesian position, executes position.concordanceInt(), assigns (vx, vy) to Cartesian velocity, executes velocity.concordanceInt(), creates Cartesian Zero.atOrigin(), and sets bool \_at\_rest to TRUE if velocity and Zero are equal, resets it to FALSE otherwise.

InertBody(double aa, double bb, double vxx, double vyy): assigns (aa, bb) to Cartesian position, executes position.concordanceDouble(), assigns (vxx, vyy) to Cartesian velocity, executes velocity.concordanceDouble(), creates Cartesian Zero.atOrigin(), and set bool \_at\_rest to TRUE if velocity and Zero are equal, resets it to FALSE otherwise.

InertBody(Cartesian pos, Cartesian vel): assigns (pos.xx, pos.yy) to Cartesian position, executes position.concordanceDouble(), assigns (vel.xx, vel.yy) to Cartesian velocity, executes velocity.concordanceDouble(), creates Cartesian Zero.atOrigin(), and sets bool \_at\_rest to TRUE if velocity and Zero are equal, resets it to FALSE otherwise.

InertBody(const InertBody &bodyCopy): (***copy constructor***) assigns (bodyCopy.position.xx, bodyCopy.position.yy) to Cartesian position of current InertBody object, executes position.concordanceDouble(), assigns (bodyCopy.velocity.xx, bodyCopy.velocity.yy) to velocity of current InertBody object, executes velocity.concordanceDouble(), creates Cartesian Zero.atOrigin(), and sets bool \_at\_rest to TRUE if velocity and Zero are equal, resets it to FALSE otherwise.

*Operator:* ***Assignment*** (operator=(const InertBody &bodyAssignment)): Same as ***copy constructor***, but checks for self-assignment, for which case nothing happens, and at the end, the current object (\*this) is returned.

*Destructor:* ~InertBody(void): deletes Cartesian position and Cartesian velocity before rest of current object is deleted.

// Read current position

int X(void);

int Y(void);

double XX(void);

double YY(void);

Cartesian Position(void);

//Set current position

void X(int a);

void Y(int b);

void X\_Y(int a, int b);

void XX\_YY(double aa, double bb);

void XX(double aa);

void YY(double bb);

void Position(Cartesian pos);

// Read current velocity

int VX(void);

int VY(void);

double VXX(void);

double VYY(void);

Cartesian Velocity(void);

bool AtRest(void);

//Set current velocity

void VX(int vx);

void VY(int vy);

void VX\_VY(int vx, int vy);

void VXX\_VYY(double vxx, double vyy);

void VXX(double vxx);

void VYY(double vyy);

void Velocity(Cartesian vel);

// Read next position

int nextX(void);

int nextY(void);

double nextXX(void);

double nextYY(void);

Cartesian nextPosition(void);

// Read previous position

int prevX(void);

int prevY(void);

double prevXX(void);

double prevYY(void);

Cartesian prevPosition(void);

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